Energy Storage for Switched Mode Power Converters

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Abstract:
The DC power converters have reactive components for steady state and transient energy storage. The energy stored in similar to the reactive components in AC system which uses a power factor defines how the energy is handled. A DC energy factor is introduced in this paper that is particular to address this issue. The set of formulation is developed to get understanding the DC energy storage and the variation of the energy stored for the inductor and capacitor.

1. Introduction

Classically, the power factor is used to express the goodness factor of an AC system. A low power factor means that the reactive power is large and large amount of energy circulates in the system. The power factor is usually connected with total harmonic distortion and the recent widely use of power electronics system also is part of the concern. On the other hand, there is no standard factor to describe the power factor in DC system. It is right that the DC system such as switched mode power converters consists of many reactive components such as inductors, capacitors and transformer. So far there are very few factors describing the energy factor in DC system. They are the for switched reluctance motor systems [1-2]. This paper is to describe the energy stored in DC/DC power converters. A set of energy factor is proposed and it is also found that the factors depend on the circuit and operation parameters.
The energy stored in the reactive components can be divided into DC and AC types. The DC type is the DC level of the current of voltage of a component. The AC value is the variation of the energy stored in the component. The following energy factors are proposed.

1.1 Storage energy for inductor

In DC/DC converters, the inductor carries both DC and AC ripples. The AC ripple is varied from the trough value I_{Lb} to peak value I_{Lt} of the current waveform with an average current $\overline{i_L}$. This variation of the energy stored in an inductor is therefore

$$S_L = \frac{1}{2} L({I_{Lt}}^2 - {I_{Lb}}^2) \qquad (1)$$

For continuous mode conduction, S can be reduced to:

$$S_L = L\,\overline{i_L}\,\Delta i_L \qquad (2)$$

where Δi_L is the peak-to-peak current ripple of the L
The ratio of the variation of the energy storage of the inductor to output power, $R_{SL,}$ is thus:

$$R_{SL} = \frac{S_L}{V_o I_o T_s} \qquad (3)$$

where V_o, I_o and T_s are the output voltage, output current and the period of the switching cycle of the DC system.
The ratio of the DC energy storage of the inductor to output power is:

$$\overline{R_{SL}} = \frac{\frac{1}{2} L \overline{i_L}^2}{V_o I_o T_s} \qquad (4)$$

1.2 Capacitor

The variation of stored energy for a capacitor C which has an average voltage of $\overline{V_C}$ and a peak to peak voltage ripple of ΔV_C can be expressed in a similar way:

$$S_C = C\,\overline{V_C}\,\Delta V_C \qquad (5)$$

The ratio of the variation of the energy storage of a capacitor to output energy R_{sc} is defined as:

$$R_{SC} = \frac{S_C}{V_o I_o T_s} \qquad (6)$$

The ratio of the variation of the DC energy storage of a capacitor to output energy $\overline{R_{SC}}$ is defined as:

$$\overline{R_{SC}} = \frac{\frac{1}{2} C \overline{V_c}^2}{V_o I_o T_s} \qquad (7)$$

The total variation of energy storage ratio for all the energy storage components is just a direct summation:

$$TESR = \sum_{all\ L,C} (R_{SL} + R_{SC}) \qquad (8)$$

2. Buck converter

2.1 Variation of energy storage

The following symbol V_{in}, V_o, I_{in}, I_o, D, Ts, L to represent the input voltage, average output voltage, average input current, average output current, transistor duty ratio, period of switching frequency respectively. The current ripple of the inductor L can be expressed as:

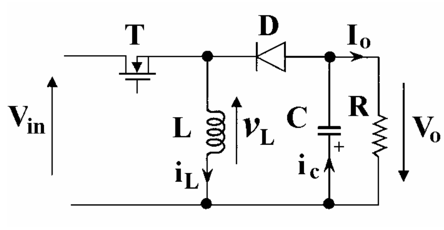
$$\Delta i_L = \frac{(V_{in} - V_o) D T_s}{L} \qquad (9)$$

Since the average inductor $\overline{i_L} = I_o$, hence, the maximum energy storage variation of the inductor is:

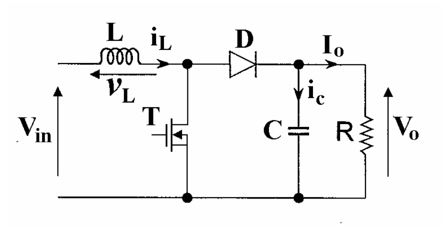
$$S_L = I_o V_o (1-D) T_s \qquad (10)$$

The energy storage variation ratio R_{SL} is thus:

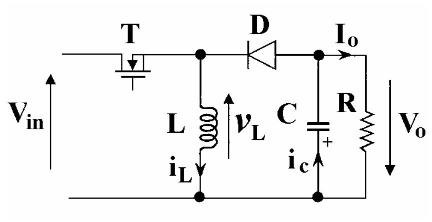
$$R_{SL} = 1 - D \qquad (11)$$



(a) Buck Converter

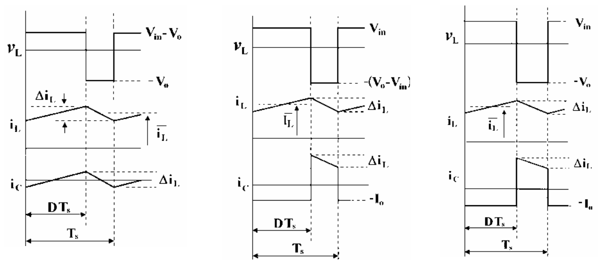


(b) Boost converter



(c) Buck-Boost converter

Fig. 1: Schematic diagram of 3 popular switched-mode converter topologies



(a) Buck Converter (b) Boost Converter (c) Buck-Boost Converter

Fig 2 Typical waveforms of the three converters under continuous inductor conduction mode

The voltage ripple of the capacitor can be obtained by considering the capacitor current as shown in Fig 2a:

$$\Delta V_o = \frac{1}{C}\int i_c\,dt \tag{12}$$

$$\Delta V_o = \frac{V_o(1-D)T_s^2}{8LC} \tag{13}$$

Hence the maximum energy storage variation of the capacitor is:

$$S_C = \frac{V_o^2(1-D)T_s^2}{8L} \tag{14}$$

The energy storage ratio for a capacitor is:

$$R_{SC} = \frac{R(1-D)T_s}{8L} = \frac{1-D}{4K} \tag{15}$$

where

K =2L / (RT_s). (16)

Therefore,

$$TESR = 1 - D + \frac{1-D}{4K} \tag{17}$$

2.2 DC energy storage

The ratio of the DC energy storage of the inductor to output power is:

$$\overline{R_{SL}} = \frac{\frac{1}{2}LI_o^{\,2}}{V_o I_o T_s} = \frac{K}{4} \tag{19}$$

The ratio of the variation of the DC energy storage of a capacitor to output energy $\overline{R_{SC}}$ is:

$$\overline{R_{SC}} = \frac{\frac{1}{2}C\overline{V_c}^{\,2}}{V_o I_o T_s} = \frac{K_c}{2} \tag{20}$$

where Kc is the normalized capacitor time constant and is equal RC/T.

3. Other converters

The energy ratio of other basic converters, namely Boost, and Buck-Boost, has been summarized in Table 1. It can be seen that they all have a simple expression of the characteristics. In the table. the $\overline{R_{SL}}$, $\overline{R_{SC}}$, and $\overline{TESR}$ are the DC energy storage ratio rather than variation of energy storage. It can be seen that $\overline{R_{SC}}$ is the same for all 3 topologies which is obvious because of the same energy stored in the output. $\overline{R_{SL}}$ is the same for Boost and Buck-Boost converter . $\overline{R_{SL}}$ for Buck converter is independent of D.

Table 2 shows the cases for isolated converter. The circuit and topologies can be referred to ref [7]. All the 6 circuits are famous topologies for used in isolated or integrated power converters.

Table 1: Energy factor for continuous mode

	R_{SL}	R_{SC}	TESR	$\overline{R_{SL}}$	$\overline{R_{SC}}$	$\overline{TESR}$
Buck	1-D	$\frac{1-D}{4K}$	$1-D+\frac{1-D}{4K}$	$\frac{K}{4}$	$\frac{K_c}{2}$	$\frac{K}{4}+\frac{K_c}{2}$
Boost	D	D	2D	$\frac{K}{4}\frac{1}{(1-D)^2}$	$\frac{K_c}{2}$	$\frac{K}{4}\frac{1}{(1-D)^2}+\frac{K_c}{2}$
Buck-Boost	1	D	1+D	$\frac{K}{4}\frac{1}{(1-D)^2}$	$\frac{K_c}{2}$	$\frac{K}{4}\frac{1}{(1-D)^2}+\frac{K_c}{2}$

Table 2: Comparison energy storage factor among 6 isolated topologies

	R_{SL}	R_{SC}	R_{SM}	TESR
Flyback	1	D	0	1+D
Forward	1-D	$\frac{1-D}{4K}$	$\frac{1}{K_M}$	$(1-D)(1+\frac{1}{4K})+\frac{1}{K_M}$
Isolated Boost	D	D	$\frac{(1-D)^2}{K_M}$	$2D+\frac{(1-D)^2}{K_M}$
Isolated Cuk	1	$1+\frac{1-D}{4K_2}$	$\frac{(1-D)^2}{K_M}$	$2+\frac{1-D}{4K_2}+\frac{(1-D)^2}{K_M}$
Boost-flyback	1+D	2D	0	3D+1
Integrated Forward	1-D	$\frac{1-D}{4K}$	$\frac{1}{K_M}$	$(1-D)(1+\frac{1}{4K})+\frac{1}{K_M}$

4. Summary of the energy factor

Fig 3a shows the TESR of the buck converter. It is expressed in terms of duty ratio D and normalized inductor time constant K. Only Buck converter is shown. The other converters are similar and will not be discussed here. It is see that the curve is linear with K >1 which is the continuous mode of inductor current. For K<1, there is condition that inductor is in discontinuous and therefore the characteristics is nonlinear.

For D = 1, the transistor is turned on 100%. TESR becomes zero. The characteristic for R_{SL}/R_{SC} is also shown in Fig 3b. The energy variation processed by L is larger than that of C as K increases which is expected. As D increases, R_{SL}/R_{SC} decreases for those discontinuous inductor conduction modes (K<0.1) that means the energy variation handled by L decreasing more than that of C.

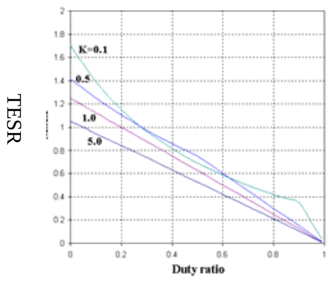


Fig 3a TESR for the Buck converter

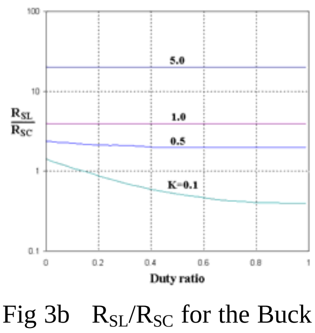


Fig 3b R_{SL}/R_{SC} for the Buck converter

Beside the characteristics of RSL, The RSC is independent of capacitance. This is because the ripple voltage on C is in general inversely proportional to C and therefore the energy stored ratio on C is independent of C but depends on L For other converters, TESR for Buck-Boost converter is always greater than 1 for all values of K and D whereas TESR for Buck converters and Boost converters is more closed to zero and less than one for continuous inductor conduction mode, therefore it is as expected that the efficiency for Buck-Boost converter is in general less than the other two. It is quite tru as compared with the general efficiency measurement for 3 converters.

5. Conclusions

This paper writes down the concept of energy storage in the passive component. Two type of expression are developed. One is the variation of storage energy TESR. The other is the DC energy storage ratio $\overline{TESR}$. The two factors are to developed some relationship to understand the energy stored in the passive components. It is expected that from there, the energy efficiency or certain control algorithm can be developed. Preliminary results have shown that they have a strong relationship to the efficiency.

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